DIFFUSION OF THERMAL PHOTONS IN THE ATMOSPHERE

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Modern theory of radiative transfer in the atmosphere is based on various modifications of Lambert's law [1]. The latter describes a flux of particles which propagates through an irreversibly absorbing medium (e.g. sunlight in water). However, it appears that application of the same approach to thermal radiation in the atmosphere is seriously flawed. Being in thermal equilibrium with the atmosphere, thermal photons cannot be irreversibly absorbed by molecules of atmospheric constituents. Absorption of thermal photons is inevitably accompanied by their emission. This has profound consequences for evaluation of the radiative properties of the atmosphere and, in particular, of the greenhouse effect and stability of the modern climate [2]. Here we show that propagation of thermal radiation in the atmosphere is governed by the random walk of thermal photons. We derive the diffusion equation describing propagation of thermal photons in the atmosphere.

The atmosphere is divided into $\tau_a = h_a/l$ layers, the distance between any two neighbouring layers being equal to the free path length of thermal photons, l. The value of h_a corresponds to the altitude of the upper radiative layer of the atmosphere, above which thermal photons pass into space unimpeded without further interaction with molecules of the greenhouse substances. On average, each k-th layer (k is counted upwards) absorbs thermal photons coming from the two neighboring layers, the upper and the lower, and thermal photons emerging locally in the course of dissipation of ordered dynamic processes in the air. In the case of isotropy all the absorbed thermal radiation is emitted with equal probability up and down. The intensities of thermal radiation I_k at each k-th layer are related to the intensities of the neighbouring layers by the condition of energy conservation [3]:

$$2I_k = I_{k+1} + I_{k-1} + A_k \quad \text{or}$$
$$-[(I_{k-1} - I_k) - (I_k - I_{k+1})] = A_k \ ,$$
$$0 \le k \le \tau_a, \quad A_k \ge 0. \tag{1}$$

Here A_k is the density of local dissipation of the dynamic energy of air per one free path of thermal photons. At the upper radiative layer $I_{\tau_e} \equiv I_e$, where I_e is the density of thermal radiation into space, which in the stationary case is equal to the intensity of solar radiation absorbed by the Earth. At the Earth's surface $I_0 \equiv I_s = I_1 + I_e$. Equation (1) is equivalent to the diffusion equation, which in the continuous form can be written as follows:

$$-\frac{d^2}{d\tau^2}q(\tau) = \delta(\tau)a_0 + a(\tau), \quad q(\tau) \equiv \frac{I(\tau)}{I_e},$$
$$a(\tau) \equiv \frac{A(\tau)}{I_e}, \quad \tau \equiv \int_0^z \frac{dx}{l(x)}. \tag{2}$$

Here $l(z) = [\sigma n(z)]^{-1}$ is the free path length of thermal radiation, $n(z)$ is the concentration of molecules of greenhouse substances, σ is the cross section of absorption of thermal radiation; variable τ is known as optical thickness of the atmosphere; a_0 characterizes direct dissipation of solar energy into thermal radiation at the Earth's surface and in the atmosphere.

The law of energy conservation with all the absorbed solar energy being transformed into thermal radiation corresponds to the following sum rule:

$$a_0 + \int_0^{\tau_a} a(\tau)d\tau = 1 \ , \quad \tau_a = \int_0^{\infty} \frac{dz}{l(z)} \equiv \frac{h_a}{l(0)} \ , \tag{3}$$

where τ_a is the optical thickness of the whole atmosphere. Solution (2), which conforms to sum rule (3), has the form

$$q(\tau) = 1 + \int_{\tau}^{\tau_a} j(\tau')d\tau',$$
$$j(\tau) \equiv -\frac{d}{d\tau}q(\tau) = a_0 + \int_0^{\tau} a(\tau')d\tau', \tag{4}$$
$$q(0) = 1 + \int_0^{\tau_a} j(\tau')d\tau' = 1 + \tau_a j(\bar{\tau}) \ ,$$
$$0 < j(\bar{\tau}) \le 1 \ , \tag{5}$$

where $j(\tau)$ is the net flux of thermal radiation.

Under condition of local thermal equilibrium the temperatures of thermal radiation and atmospheric air, T, coincide. Then $q(\tau) = \left(\frac{T(\tau)}{T_e}\right)^4$ and the vertical gradient of air temperature (lapse rate) $dT/dz = \frac{1}{l(z)}\frac{T}{4q}\frac{dq}{d\tau} \approx -\frac{T_s}{4l(h_a)} \approx 6.4°C/km$. This estimate approximately coincides with the observed value (6.6°C/km), it is independent of the acceleration of gravity and vanishes at $l(h_a) \to \infty$, i.e. in the absence of interaction between the greenhouse gases and thermal radiation.

When the dynamic processes in the atmosphere, and, consequently, processes of their dissipation, are relatively small, $a_0 \approx 1$, $a(\tau) \ll a_0$, the net flux of thermal radiation is of the order of unity and does not change with time, despite that the rate of decrease of intensity of thermal radiation, which is equal to the net flux of radiative thermal energy, has a non-zero value. This is in contrast to predictions of the traditional radiative transfer equation. As is clear from (5) and (3), dissipation of the dynamic energy of the air decreases the intensity of thermal radiation at the surface and, consequently, the greenhouse effect, but cannot make the latter equal to zero. (Greenhouse effect is quantified as the difference between the intensities of thermal radiation at the surface and at the upper radiative layer of the atmosphere).

In the case of several, $1 \leq i \leq N$, greenhouse substances, each of them having an absorption band in the spectral interval $\Delta\omega_i$, the normalized intensity of thermal radiation at the Earth's surface has the form:

$$q(0)\delta_i = (1 + \tau_{ai})a_i, \quad q(0) = \frac{1}{b}, b \equiv \sum_{i=1}^{N} \frac{\delta_i}{1 + \tau_{ai}}; \quad (6)$$

$$\delta_i \equiv \frac{\delta\omega_i I_P(\omega_i, T)}{I_s}, \quad a_i \equiv \frac{I_{ei}}{I_e},$$

$$\sum_{i=1}^{N} a_i = \sum_{i=1}^{N} \delta_i = 1 ,$$

where δ_i is relative share of the equilibrium thermal radiation of the Earth's surface, $I_s = \sigma T^4$, which corresponds to the spectral interval of the absorption band of the i-th greenhouse substance, $I_P(\omega_i, T)$ is the Planck distribution; a_i is the relative share of the thermal radiation of the upper radiative layer of the atmosphere in the same spectral interval; b has the meaning of effective transmissivity of the atmosphere with respect to thermal radiation of the Earth's surface.

The major contribution into terrestrial greenhouse effect is made by water vapour, CO_2 and cloudiness, the latter absorbing thermal radiation rather evenly over the whole thermal spectrum. Thus, we have for the transmissivity of the terrestrial atmosphere:

$$b = \frac{\delta_{H_2O}}{\tau_{a0} + \tau_{aH_2O} + 1} + \frac{\delta_{CO_2}}{\tau_{a0} + \tau_{aCO_2} + 1} + \frac{\delta_0}{\tau_{a0} + 1}. \quad (7)$$

Here δ_0 is the share of thermal radiation corresponding to a spectral interval where there are no other absorbers except for various forms of non-gaseous water (spectral window). Variable τ_{a0} is the sum of the mean optical thickness of cloudiness and the mean optical thickness of the oceanic water layer penetrable by the sunlight. We note that the latter is likely to be larger than the former. In such a case, the greenhouse effect under the oceanic surface is mainly controlled by the average depth of absorption of solar radiation by the oceanic water, which is determined by oceanic densities of organic substances. In the clean water the optical depth of the corresponding oceanic layer would be much larger than that of the atmosphere. The surface temperature of such a "clean water" ocean could then approach the modern temperature of the upper radiative layer of the atmosphere, i.e. about $-18°C$, which is below the freezing point of water. Eddy mixing of the subsurface layer of the ocean works to diminish the temperature interval between the surface and the average depth of absorption of solar radiation. Regulation of the density of dissolved and suspended organic substances in the ocean makes it possible to keep the oceanic surface temperature at any value between the average land surface temperature and that of the upper radiative layer of the atmosphere. It follows that the oceanic biota maintains the very existence of liquid ocean by regulating the turbidity of the oceanic water.

According to the definition, τ_{ai} (3) is proportional to the average concentration n_i of the i-th greenhouse substance. Thus, formulae (6), (7) specify the dependence of the normalized intensity of thermal radiation at the surface, $q(0)$, and greenhouse effect, $q(0) - 1$, on concentrations of greenhouse substances.

References

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